NEW CONCEPT

MATHEMATICS

for Senior Secondary Schools

Teacher's Guide

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First published 2012
Second edition 2014
Third edition 2016
Reprinted 2019
Fourth edition 2024

ISBN 978 978 925 826 0

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Chapter 1 Logarithms

Objectives

By the end of this chapter, the students will be able to:

1 find the logarithm of numbers less than 1, using mathematical tables;
2 use the logarithm table to solve problems with negative characteristics;
3 use logarithm tables to carryout calculations involving powers and roots of numbers less than 1; and
4 Solve simple equations involving logarithms.

Logarithms of numbers greater than 1 (Page 11)

Give students the definition of the logarithm of a number which is of two parts, namely: the characteristic or integer and the mantissa. The characteristic or the integer is a whole number which could be positive or negative while mantissa is a decimal number which is positive.

Quickly revise with students how to use tables to calculate the logarithms of numbers greater than one by going through Examples 1–3 on pages 11 and 12.

Exercise 1.2 (Pages 12 and 13)

Give some questions as classwork and the rest as an assignment.

Expressing numbers less than 1 in standard form (page 13)

Lead the students to give examples of numbers less than one, then ask them to write those numbers in their standard form.

Guide the students on how to obtain the characteristic or the integer of a given number. The characteristic or the integer can be determined or obtained by expressing the given number in standard form ($A \times 10^n$), where the power of 10 which is n is the integer or characteristic of the given number.

For example, find the logarithm of 0.000 592.

In standard form, 0.000 592 = 5.92×10^{-4} and integer or characteristic is $\bar{4}$.

$\therefore$ log 0.000 592 = $\bar{4}.7723$.

Exercise 1.3 (Page 14)

Give the students Questions 1–5 (Page 14) as a drill.

Give the students Questions 9–12 (Page 14) as a classwork, and supervise them. Mark and give the necessary corrections.

Give the students Questions 7, 8, 13, 14 and 15 (Page 14) as an assignment.

Logarithms of numbers less than 1 (Page 14)

Guide the students on how to add, subtract, multiply and divide the integers or the characteristic of numbers less than one, using Examples 5 to 7 on pages 15 and 16.

Exercises 1.4 and 1.5 (Pages 15 and 16)

Give the questions in Exercise 1.4 as classwork and Questions 2 – 15 of Exercise 1.5 as an assignment.

Powers and roots of numbers less than 1 (Page 16)

Guide the students in calculations involving multiplication, division, powers and roots of numbers less than one and let the students realise that the same rules used in solving numbers greater than one are also applicable to the logarithm of any number less than one.

For example:

Evaluate

a) $(0.925)^3$ **b)** $\sqrt[4]{0.0764}$

a)

No.	Log
$(0.925)^3$	$\bar{1}.9661$
	$\times \quad 3$
(0.7912	$\bar{1}$**.8983**

(+2 is carried to the integer or characteristic part: $\bar{1} \times 3 = \bar{3} + 2 + \bar{1}$)

b)

No.	Log
$(0.0764)^{\frac{1}{4}}$	$\bar{2}.8831 \div 4$
$=$	$\dfrac{\bar{4} + 2.8831}{4}$
0.5258	$\bar{1}$**.7208**

($\bar{2}$ is not divisible by 4 easily, therefore, replace $\bar{2}$ with $\bar{4} + 2$, so that the negative part would be divisible by 4.)

Guide the students on how to place the decimal point after taking the antilogarithm.

From the example above, add 1 to the integer, $\bar{1} + 1 = 0$.

That means, after the decimal point, write 5258.

Assuming we have $\bar{3}.2563$, from the antilog table, $2563 = 1804$

Add 1 to the integer, $\bar{3} + 1 = \bar{2}$.

That means, after the decimal point, write two zeros before 1804.

$\therefore$ antilog $\bar{3}.2563 = 0.00\ 1804$.

Discuss Examples 8 and 9 on page 17 and give the students questions to solve on their own especially previous *WAEC* or *NECO* questions.

Exercise 1.6 (Page 18)

Give the students Questions 11 – 15 as classwork and Questions 1 – 10 as an assignment.

Solving simple logarithmic equations (Page 18)

Make the students understand that the relationship between indices and logarithm has made it possible to convert from index notation to the logarithmic form and vice versa. The relationship can be used to solve simple logarithmic equations.

Exercise 1.7 (Page 18)

Give the students Question 1 – 8 as classwork and the rest as an assignment.

Chapter 2 Approximation

Objectives

By the end of this chapter, the students will be able to:

1 round off numbers to the nearest tens, hundreds, thousands, whole numbers decimal place, and significant figures;

2 approximate and estimate calculations close to the correct answer;

3 determine the degree of accuracy of results;

4 find the percentage error in a measurement; and

5 apply approximation in everyday-life activities.

Discuss approximation, rounding off significant and decimal places with the students using Examples 1–5 (Pages 20–22).

Rounding off (Page 20)

Rounding off is a way of approximating numbers. When a number is large and cannot be easily handled, we round it off in order to handle it. Numbers 0, 1, 2, 3 and 4 are rounded down to 0, while numbers 5, 6, 7, 8 and 9 are rounded up to 1.

Rounding to decimal places (Page 21)

Decimal means a dot written between the unit figure of a number and its fractional part when expressed in decimal system e.g. 54.847 = 54.85. The last digit; 7, is rounded up to 1 and added to 4. There are two digits after the decimal point i.e. 54.847 = 5485 to 2 decimal places.

Significant figure (Page 22)

This is a number which makes a contribution to a value. Significant figures are used to denote the accuracy of an answer. If zero is in between any given number. It cannot be counted as significant. For example, in 0.005477, the first significant figure is 5, which is more significant compared to 4.

Exercise 2.1 (Page 23)

Give the students Questions 1–8 (Page 18) orally in the class.
Guide the students to differentiate between the results obtained from solution using logarithm tables and using calculators, as in Examples 6–8 on page 22.

Approximation (Page 23)

An approximation is a number taken as close as possible to the actual value of the number. Approximation gives a result very close to the answer, but not the exact answer.

Exercise 2.2 (Page 24)

Give the students Questions A and B (Page 4) as a classwork and Question C as an assignment using Examples 9–14 to explain.

Percentage and relative error (Page 26)

Lead the students on how to calculate percentage error by giving them formulae for percentage error and relative error.

$$\text{Relative error (R.E.)} = \frac{\text{maximum absolute error}}{\text{absolute value}}$$

$$= \frac{\text{precision}}{\text{measurement}}$$

$$\text{Percentage error (P.E.)} = \frac{\text{maximum absolute error}}{\text{measurement}} \times 100\%$$

$$= \frac{\text{error} \times 100\%}{\text{true value}}$$

Lead the students through Examples 15–17 (Pages 26 and 27) for easy understanding.

Exercise 2.3 (Page 27)

Encourage the students to solve the questions as classwork. Discuss the questions and additional examples with them to promote exchange of ideas among the students in the class.

Degree of accuracy (Page 28)

Explain to the students using Examples 18 – 20 (page 28) that in calculations involving addition of data, the degree of accuracy is the sum of the respective approximation, meaning that the precision of a sum is equal to the sum of the precision of the measurements.

Exercise 2.4 (Page 29)

Give the students Questions 1 – 8 as classwork and rest as an assignment.

Application of approximation to real-life situations (Page 29)

Use Examples 21 – 23 (pages 29 and 30) to show the use of approximation in real-life situations.

Exercise 2.5 (Page 30)

Give the questions in this exercise as class test.

Chapter 3 Sequences and series

Objectives

By the end of this chapter, the students will be able to:

1 identify sequences and series, and differentiate between them;
2 state the rules of any given sequence and series;
3 find the terms of any given arithmetic progression (AP);
4 find the nth term and the sum of an arithmetic progression;
5 define a geometric progression (GP) in terms of common ratio r and first term q;
6 calculate the nth term of a geometric progression and the sum of a finite GP; and
7 calculate the sum of a GP to infinity.

Sequence (Page 31)

A sequence is an ordered list of numbers, obtained by a certain rule. The rules may differ depending on the type of sequence.

For example:

a) 1, 2, 3, 4, 5… $n + 1$: $n = 0, 1, 2\ldots$
b) 3, 6, 9, 12, …3^n; $n = 1, 2, 3, \ldots$
c) 2, 4, 8, 16, …2^n; $n = 1, 2, 3, \ldots$

Lead the students to discover the different types of sequence. Write sequences that have terms increasing or decreasing in equal steps and ask the students to give the rules that guide the sequence or pattern.

Exercise 3.1 (Page 32)

Give the students the Questions orally in the classroom.

Exercise 3.2 (Page 33)

Lead the students step by step on how to generate the rule and give them Questions in this exercise as a classwork.

Exercise 3.3 (Pages 33 and 34)

Give the students Question A to be done orally in the classroom and Questions B and C as assignment.

Series (Page 34)

A series is the sum of terms of a given sequence. The following are examples:

a) $1 + 2 + 3 + 4 + \ldots + 100 + 101 + 102$
b) $2 + 4 + 8 + 10 + \ldots$

Series are classified into two, namely finite and infinite.
A finite series is a series that has an end while an infinite series is series that is endless.

Arithmetic progression (AP) (Page 35)

Lead the students to define an arithmetic progression, determine first term (a) of a sequence as well as to obtain common difference (d).
A sequence whose term either increases or decreases in equal steps is called an arithmetic progression (AP) e.g. $x, x + 1, x + 2 \ldots$
From the example above,
x is the first term of the sequence. 2nd term minus first term gives the common difference.
$\therefore$ common difference d of the given sequences will be second term – first term = 3rd term – 2nd term, i.e.,

$$\begin{aligned} x + 1 - x &= x + 2 - (x + 1) \\ x - x + 1 &= x + 2 - x - 1 \\ 1 &= x - x + 2 - 1 \\ 1 &= 1 \end{aligned}$$

Exercise 3.4 (page 34)
Give the students Questions 1– 5 as classwork and Questions 6 – 10 as assignment.

The nth term of an arithmetic progression (Page 36)

Guide the students to derive the nth term of an AP from the given sequence and sum of terms of an AP. Use examples to discover how $S = \frac{1n}{2}(a + (n - 1)d)$ is derived from $S = \frac{1}{2}n$ (a+1).

Exercises 3.5 and 3.6 should be discussed with the students after treating Examples 1–7 (Pages 32–36). Encourage the students to attempt all the previous WAEC and JAMB/UTME questions in those exercises.

Sum of terms of an AP (page 37)

Explain this to students using Examples 8 – 10 (pages 38 and 39) and give them some questions in Exercise 3.7 (page 39) as classwork and the rest as an assignment.

Geometric progression (GP) (Page 39)

The students should be able to understand the meaning of geometric progression (GP), determine first term a of a GP and how to obtain the common ratio r of a given GP.
Explain to students that a sequence in which the terms increase or decrease in equal common ratio is called geometric progression, e.g.

a) $3a, 6ab, 12ab^2,$ ________

b) 8, 4, 2, ________

From the examples above,
first term (a) of a is $3a$
first term (a) of b is 8.

$$\text{common ratio } r = \frac{\text{2nd term}}{\text{1st term}} = \frac{\text{3rd term}}{\text{2nd term}}$$

$\therefore$ **a)** $r = \dfrac{6ab}{3a} = \dfrac{12ab^2}{6ab} = 2b$

b) $r = \dfrac{4}{8} = \dfrac{2}{4} = \dfrac{1}{2}$

Exercise 3.8 (Page 40)
Give the students Questions 1 – 5 to be done in class while you go round to check. And give Questions 6 – 10 as an assignment

The n^th term of a GP (Page 40)

Lead the students to derive the nth term of a GP from the sequence.

nth of a GP is defined by $Tn = ar^{n-1}$

Discuss any difficult questions in Exercise 3.6 in the classroom and ensure that the students attempt all the questions most especially previous WAEC and JAMB/UTME questions.
Explain when to use the two formulae for calculating the sum of a GP, i.e.

$$Sn = \frac{a(r^n - 1)}{r - 1} \quad \text{Iff } r > 1$$

or $$Sn = \frac{a(1 - r^n)}{l - r} \quad \text{Iff } r < 1$$

The sum of a GP to infinity (Page 43)
Guide the students to calculate the sum of a GP to infinity, using the formula: $S_\infty = \dfrac{a}{1 - r}$.

Encourage the students to attempt Questions 1–3 and 8–10 in Exercise 3.10, page 43 and discuss all the questions in Exercise 3.11 (Page 45) with them in the class.
Give them Questions 4–9 as an assignment.

Chapter 4 Quadratic equations

Quadratic equations

Objectives

By the end of this chapter, the students will be able to:

1 factorise quadratic perfect squares completely;
2 make quadratic expression a perfect square;
3 solve quadratic equations by factorisation method;
4 solve quadratic equations by method of completing the square;
5 derive and apply the quadratic formula in solving quadratic equation problems;
6 form quadratic equations from the sum and product of the given roots; and
7 solve word problems leading to quadratic equations.

Factorisation of perfect squares (Revision) (Page 46)

Revise with the students, the two methods of solving quadratic equations, which are: factorisation and graphical methods.

Discuss with the students that difference of two squares and perfect squares are only special cases of quadratic equations. Using Examples 1 and 2 (Pages 46 and 47), assist the students to identify a perfect square quadratic expression.

Exercise 4.1 (Page 47)

Give the students the odd numbered questions as classwork. Supervise and mark. Give them the necessary corrections.

Give them even numbered questions as homework.

Making quadratic expressions into perfect squares (Page 47)

Guide the students on how to make non-perfect squares quadratic expressions to be perfect by stating the procedure involved. Lead the students to realise that all perfect square quadratic expressions are factorisable.

Exercise 4.2 (Page 48)

Lead them through the Examples 3-5 and give them 13–20 (Pages 47 and 48) as classwork. Supervise and mark their answers.

Solving quadratic equations by completing the square method (Page 48)

Discuss with the students, the steps involved in solving quadratic equations that are not factorisable, using completing the square method as in Examples 6 and 7 (Page 49).

Exercise 4.3 (Page 50)

Give the students Questions 12–20 (Page 50) to solve as classwork and the rest as homework.

Quadratic equations with irrational roots (Page 50)

Ensure that the students understand how to solve equations with irrational roots, using Example 8 (Page 50).

Exercise 4.4 (Page 50)

Give the students Questions 5–12 as classwork and 13–20 as homework.

Derivation of quadratic formulae from completing the square (Page 51)

Lead the students to discover how to derive the quadratic formula, by solving the quadratic general form using completing the square method.

$$x = -\frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

Demonstrate to the students how to apply the formula to solve quadratic equations using Examples 9 and 10 (Pages 52 and 53).

Deriving quadratic equations from the sum and products of roots (Page 53)

Discuss with the students how to use the sum and product of quadratic equations to solve problems. For example, $ax^2 + bx + c = 0$ is a quadratic equation which can be written as

$$ax^2 - (u + v)x + uv = 0 \text{ and } x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Comparing the coefficients of x and constant c, $-(u + v) = \frac{b}{a}$ × sum of roots (i.e. u and v)

Hence, $u + v = \frac{b}{a}$ and $uv = \frac{c}{a}$ × product of roots (i.e. u and v)

Explain how to construct a quadratic equation from the given roots using Examples 11 and 12 (Pages 53 and 54).

Word problems leading to quadratic equations (Page 54)

Guide the students on how to form quadratic equations from word problems, using Examples 13, 14 and 15 (Page 55).

Encourage the students to solve Exercises 4.5, 4.6 and 4.7 (Pages 53–56).

Chapter 5 Simultaneous linear and quadratic equations

Objectives

By the end of this chapter, the students will be able to:

1 solve simultaneous linear equations;
2 solve a pair of equations, one linear and one quadratic, analytically;
3 solve a pair of equations; one linear and one quadratic, graphically;
4 apply the principles of the solutions of simultaneous and quadratic equations to related problems; and
5 solve word problems leading to simultaneous, linear and quadratic equations.

Simultaneous linear equations (Page 57)

Explain to the students that simultaneous equations are a set of two algebraic equations that are both true for the same particular values of their variables when solved at the same time. When the equations involved are both linear, they are known as simultaneous linear equations which can be solved by three methods, namely:

1 elimination method;
2 substitution method; and
3 graphical method.

Revise the three methods of solving simultaneous linear equations with the students using Examples 1–5 (Pages 57–59) and encourage them to solve Exercise 5.1 (Pages 59 and 60) most especially, previous WAEC and JAMB/UTME questions in the exercise. Let the students realise, that fractions should first be cleared when solving simultaneous equations involving fractions.
Supervise and mark the classwork and give the necessary corrections.
Give part of the questions as homework.

Simultaneous linear and quadratic equations (Page 60)

Lead the students to solve simultaneous equations involving one linear and one quadratic equation analytically using the substitution method. Discuss Exercise 5.2 (Pages 60–61) Questions 11 and 18 with the students in the classroom. Give the Questions 12–16 as classwork. Also, give them Questions 9, 17, 19, and 20 as assignment.

Graphical solution of simultaneous linear and quadratic equations (Page 61)

Guide the students on how to compute tables of values of y given the values of x and to plot the points from the tables of values on the Cartesian plane i.e. x-axis and y-axis.

The linear equation, $y = bx + c$ gives a straight line graph while the simultaneous equations ($y = ax^2 + bx + c$) gives either cup-shaped or cap-shaped curves depending on the coefficient of x^2, as in Examples 8 and 9 (Pages 61 and 62). Exercise 5.3 (Page 63), Questions 1, 2 and 8 to be given as classwork.

Solving related problems involving graphical solutions of linear and quadratic equations (Page 63)

Assist the students to read the points of inter-section of the curve and the straight line which is the solution of the simultaneous equation.

Give Exercise 5.4 (Page 63) orally in the classroom and Exercise 5.5 on pages 66 and 67, WAEC and JAMB/UTME questions as classwork.

Word problems involving simultaneous, linear and quadratic equations (Page 67)

Guide the students to solve word problems leading to simultaneous equations using Examples 13–15 (Pages 68 and 69). Discuss Exercise 5.6, Questions 2, 3, 5, 8 and 10 with the students in the class.

Chapter 6 Gradient of a curve

Objectives

By the end of this chapter, the students will be able to:

1 review the graph of $y = mx + c$ and draw the graph of linear equation of the form $ax + by + c = 0$;

2 identify the x– and y– intercepts of any given linear equation and plot a straight line from the intercepts;

3 find the gradient of a straight line;

4 construct the equation of a straight line:

- from a gradient and one point;
- through two points; and

5 draw the tangent to a curve at a given point and calculate its gradient.

Explain to the students that gradient is the amount of inclination of a line or a curve at a particular point. The gradient of a curve at a given point is the slope of the tangent to the curve at that point. Show the students how to calculate the gradient of lines using graph and calculation.

Straight line graph (Page 70)

Give the general form of a straight line equation as $ax + by = c$.

Explain x and y-intercepts, using Example 2 (Pages 71 and 72) and encourage the students to attempt all the questions in Exercise 6.1 (Pages 72 and 73). Supervise and mark. Then, give the necessary corrections.

Gradient of a straight line (Page 73)

Discuss the two methods of calculating the gradient of a curve.

The gradient of a line is generally represented by m.

Hence, the gradient of any line $(m) = \dfrac{\text{vertical change}}{\text{horizontal change}}$

$$= \frac{\text{increase in } y}{\text{increase in } x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

The equation of a line in the gradient intercept form is $y = mx + b$ where m is the gradient and b

$(0, b)$ is the y-intercept.
Discuss Examples 3–5 with the students, ensure that the students attempt Questions 1–5 and 11–15 in Exercise 6.2 in the classroom. Give the students 6–11 as assignment. Mark and give the corrections.

Equation of a straight line (Page 75)

Guide the students on how to find the equation of a straight line from point gradient form and equation of a line through two points using Examples 6 and 7 (Pages 75 and 76). Discuss Exercise 6.3 (Page 69) Questions 4 and 10 with the students in the classroom. The equation of the line through a point (x_1, y_1) and (x_2, y_2) is given as $\frac{y_2 - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

Gradient of a curve at a given point (Page 76)

Lead the students to realise that the gradient of a curve at a given point can be found by drawing a tangent to that given point and that the gradient at turning point of a curve is zero (0). Guide the students on how to obtain the minimum and maximum points of a curve as well as the line of symmetry using Examples 8 and 9 (Page 77).

Remind them that they should construct a table of values and follow the given scale, but if the scale is not given, they should choose a suitable scale.
Ensure that the students attempt Exercise 6.4 Questions 4–7 (Page 78) as a classwork. Supervise them and mark. Give them the necessary corrections.

Chapter 7

Logical reasoning

Objectives

By the end of this chapter, the students will be able to:

1 give examples of simple and compound statements;

2 list the five logical operations and their symbols;

3 write the truth value of a compound statement involving any of the five logical operations;

4 use the truth table to prove that:

a) a contra-positive is equivalent to the conditional statement;

b) a converse is equivalent to an inverse of a conditional statement; and

5 apply contra-positive and inverse statement in proving theories.

Statements; proposition (Page 84)

Revise the previous lesson with the students. Explain that a simple statement is a statement which has only one main verb, while the compound statement is a statement made up of two or more simple statements which is also known as composite statement. Give Exercise 7.1 page 84 and Exercise 7.2 page 84 as classwork.

Draw a table showing connectives and their notations. Give some examples of simple and compound statements and guide the students to identify which of the statements is simple and which one is compound.

Lead the students to compute the truth table. For example:

Conjunction

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Here, the two statements must be true i.e.

T and T → T

T and F → T

and F and F → F

Disjunction

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Here, either the first statement or the second is true i.e.

T or T → T

For T or F → T

or F or F → F

Negation

p and ~p

P	~P
T	F
F	T

Conditional statements

p	q	p⇒q
T	T	T
T	F	F
F	T	T
F	F	T

Bi-implication

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

a) T ↔ T ↔ T and F ↔ F ↔ T
b) T ↔ F ↔ F and F ↔ T ↔ F

Give Exercise 7.3 (Page 87) to students to do as their classwork.

Conditional statements (Page 87)

Guide the students to state the converse, inverse and contra-positive of a given conditional statement.

For example, *if two angles are alternate angles then they are equal* is a conditional statement. *If two angles are equal then they are alternate angles* is a converse statement. But *if the two angles are not equal then they are not alternate angles* is an inverse statement or contra-positive is equivalent to conditional statement and converse is equivalent to inverse of a conditional statement using Examples 7 and 8 (Page 89).

Tautology and contradiction (Page 90)

Use Examples 9 and 10 (Page 90) to explain tautology and contradiction of a compound statement.

Laws of algebra of logical statements (Page 91)

Discuss laws of algebra, for logical statements with students in the classroom.

Chapter 8

Linear inequalities

Objectives

By the end of this chapter, the students will be able to:

1 solve inequalities in one variable;
2 solve inequalities in two variables;
3 determine the range of values of combined inequalities;
4 draw graphs of linear inequalities in two variables;
5 obtain the solution's sets, or regions which satisfy simultaneous inequalities;
6 solve word problems involving inequalities; and
7 apply linear inequalities to programming.

Linear inequalities in one variable (Page 93)

Inequality is a mathematical statement in which one quantity is greater or lesser than the other.

Revise linear inequalities in one variable with the students. Draw a table showing the symbols that are commonly used in inequality. Explain how to sketch the solutions of linear inequalities on a number line using Examples 1(a-e) (Pages 93 and 95). Let the students attempt Questions 1, 2, 4, 6, 7, 9 and 10 in Exercise 8.1 (Page 95) in the class as classwork.

Combining inequalities (Range of values) (Page 96)

Guide the students to realise that compound statements can be combined to form a single inequality. For example:

x is greater than –2 and x is less than or equal to 5 $\Rightarrow -2 < x \leq 5$

This statement implies that x is between –2 and 6 on the number line. It is represented thus:

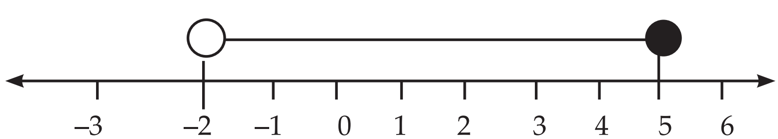

Fig. 8.1

From Fig. 8.1, the empty circle O indicates that –2 is not included in the values of x and the shaded circle also indicates that the value 5 is included in the values of x.

$\therefore$ the values of x include: –1, 0, 1, 2, 3, 4, 5. (Exercise 8.4 (Page 11) Questions 1-5 can be answered orally, but give Questions 6-20 to the students as classwork.

The cantesian plane (Page 101)
Lead the students to construct tables of values and guide them on how to plot the values on the Cartesian graph. Then, explain how to determine the region that belongs to the set of any given inequalities in two variables on the Cartesian graph and how to obtain the required region, which always pose problems to students. From Example 6 (Page 102), let the students realise that the broken line shows that the points along the line are not included. Discuss Example 7 (Pages 102–103) with the students.

Simultaneous inequalities (Page 104)
Use Examples 8 and 9 (Pages 104 and 105) to explain how to solve simultaneous inequalities in two variables. From the Cartesian graph, let the students realise that the unshaded region within the given simultaneous inequalities is the solution. Ensure that the students attempt Questions 2 and 7 (Page 94) in Exercise 8.6 (Page 106).

Application of inequalities to real-life situations (Linear programming) (Page 106)
Guide the students on how to apply linear inequalities to real-life situations using Examples 10–11 (Pages107 and 109). Give the students Questions 1 and 2 of Exercise 8.7 (page 109) as classwork.

Lead the students on how to apply linear inequalities to linear programming in day-to-day activities. Give them Exercise 8.7 (Pages 109 and 110), Questions 3, 5, and 6 and Exercise 8.8 (Page 111), Questions 3, 4 and 5 (Page 111) to do as their classwork or an assignment.

Chapter 9 Algebraic fractions

Objectives

By the end of this chapter, the students will be able to:

1 reduce algebraic fractions to their lowest terms;

2 simplify algebraic fractions involving addition, subtraction, multiplication and division;

3 solve simple equations involving algebraic fractions;

4 solve simultaneous equations involving algebraic fractions;

5 determine the value of the unknown, when an algebraic fraction is undefined.

Simplification of algebraic fractions (Pages 101 and 102)

Algebraic fraction is a part of a whole represented mathematically by a pair of algebraic terms.

Examples of algebraic fractions include the following: $\frac{a}{ab}, \frac{xyz}{wxz}, \frac{9x^2 - y^2}{y^2 - 2xy - 3x^2}$, etc.

To simplify algebraic fractions, we need to factorise both the numerator and the denominator. It is then possible to cancel out the common factors.

Operations in algebraic fractions (Page 115)

Let the students realise that to add or subtract algebraic fractions, they have to find the LCM of the denominators of the fractions and that algebraic fractions can be reduced to their lowest terms. Discuss Examples 1–7 (Pages 113–116) with the students and give them questions to do in Exercises 9.1 and 9.2 (Pages 115 and 116).

Use Example 8 (Page 117) to explain changes in the sign.

Note that:

$(y - x) = -(x - y)$

$(p - q) = -(q - p)$

When dividing or multiplying algebraic fractions, students should factorise both the numerator and denominator and cancel out the common factors where necessary. Guide the students through Examples 9 and 10, (Page 117) for easy explanation and understanding. Let the students attempt Questions 7, 12 and 20 in Exercise 9.3 (Pages 117 and 118) as classwork.

Equation involving fractions (Page 118)

Guide the students to solve equations involving fractions using Examples 11 and 12 (Page 118). Discuss Questions 10, 13, 19 and 20 with the students in Exercise 9.4 (Page 119). Give them as a classwork first before discussing.

Substitution in fractions (Page 119)

Lead the students to attempt how to substitute value or number for the unknown in a given fraction using Examples 13 and 14 (Page 120). Ensure that the students attempt Questions 4, 6 8 and 10 (Page 120) of Exercise 9.5 as classwork and Questions 1, 5, 7 and 9 as homework.

Simultaneous equations involving fractions (Page 120)

Lead the students to simultaneous equation involving fractions. Discuss Examples 15 and 16 (Pages 120 and 121) with the students and give them Questions 1, 2 ,5 and 10 in Exercise 9.6 (Page 121) and the rest as assignment.

Undefined algebraic fractions (Page 121)

Guide the students to determine undefined value of a fraction. Let the students realise that a fraction is undefined when the denominator is zero. For example, $\frac{x+2}{x^2-3x-10}$ is undefined when the denominator $(x^2-3x-10)$ is equal to zero.,

i.e. $x^2 - 3x - 10 = 0$

then solve for x by factorising the left hand side of the equation.

$x - 5x + 2x - 10 = 0$

$x(x-5) + 2(x-5) = 0$

$(x+2)(x-5) = 0$

Either $x + 2 = 0$ or $x - 5 = 0$

$x = -2$ or $x = 5$

The fraction is undefined when $x = 5$ or -2.

$\frac{x+2}{x^2-3x-10}$ Substituting 5 into the fraction, we have:

$\frac{x+2}{5^2-3\times5-10} = \frac{7}{25-15-10} = \frac{7}{25-25} = \frac{7}{0}$ (Any number divided by zero is undefined.)

Ensure that the students attempt Questions 8, 13, 14 and 15 in Exercise 9.7 (Page 123) in the class and give the rest as an assignment.

Chapter 10 Chord properties of a circle

Objectives

By the end of this chapter, the students will be able to:

1 define an arc, a chord, a segment and a sector of a circle;
2 determine the angle subtended by a chord of a circle;
3 solve problems on angles subtended by chords and perpendicular bisectors of chords; and
4 solve problems on alternate line segment.

Chord properties of a circle (Page 125)

Guide the students to define chord and chord properties of a circle.

A **circle** is a perfect round plane contained by a line, which is equidistant from a fixed point within it.

A **chord** is a straight line that joins two points on a circle.

A **diameter** is the length of a straight line that bisects a circle. It passes through the centre of the circle and is equal to twice the radius, i.e. $D = 2r$.

A **radius** is a straight line from the centre of the circle to any part of the circumference of a circle.

A **segment** is part of a circle bounded by an arc and chord. The larger part of the circle is called the major segment and the smaller part of the circle is called **minor segment**. Each of these parts is called the **alternate segment** of the other. Ask the students to draw a large circle and label radius, chord, arc, major segment and minor segment.

Lead the students, instructing models to show angles subtended by chord at the centre; perpendicular bisectors of chords and angles in alternate segments using Activities 10.1 and 10.2(Page 126). Discuss Examples 1–2 (Pages 126 and 127). Encourage the students to attempt at least five questions out of ten questions in Exercise 10.1 (Page 127) and give the rest as an assignment.

Chord theorems (Page 128)

Guide the students to carry out formal proofs of the riders one after the other and give examples on the riders. Explain result, using practical models constructed. Make sure that you go through the activities with the students since it will help them to understand the theorems better and go through the examples as well. The examples will enable the students to be familiar with types of questions and approach of solving problems.

Give the students Exercise 10.2 (Pages 130 and 131) to do as assignment and discuss all the previous *WAEC* questions.

Chapter 11 Circle theorems

Objectives

By the end of this chapter, the students will be able to:

1 prove that the angle which an arc of a circle subtends at the centre is twice the angle it subtend at the circumference;
2 prove that angles on the same segment of a circle are equal;
3 prove that an angle in a semi-circle is a right-angle;
4 prove that opposite angles in a cyclic quadrilateral are supplementary;
5 prove that exterior angle is equal to the interior opposite angle;
6 prove riders on tangent to a circle; and
7 solve practical problems on circle throrems.

Review the format for proving Euclidean geometry such as:

Given: ______________________________

Required to be proved: ______________________________

Construction: ______________________________

Proof: ______________________________

Q.E.D: ______________________________

Angles at the centre of the circle (Page 123)

State theorem 1 of circle theorems and lead the students to demonstrate the theorem using framators and models. Draw the diagram of the circle theorem. Guide the students on how to prove theorem one by asking them to suggest reasons why certain conclusions should hold. It is advisable to revise the theorem on the exterior angle of a triangle equal to the sum of opposite interior angles. Discuss Examples 1 and 2 (Pages 133 and 134) with them and give them all the previous WAEC questions in Exercise 11.1 (Pages 134 –136) to do as classwork. Encourage the students to state reasons for their answers.

Angles in the same segment of a circle (Page 136)

State theorem 2 and theorem 3. Theorem 1 is a pre-requisite to the two theorems. When explaining the theorems, always refer to theorem 1. Demonstrate the solution of practical problems leading to the theorem. Ensure that the students solve Question 1 and all the previous WAEC questions in Exercise 11.2 (Pages 137 and 138). Give the rest as an assignment.

Cyclic quadrilaterals (Page 138)

Guide the students to define a quadrilateral and give examples. Let the students realise that a quadrilateral inscribed in a circle with all its vertices on the circumference is known as a **cyclic quadrilateral**. Discuss Examples 5 to 11 (Pages 139–142) with them and give them Exercise 11.3, Questions 2, 3, 6 and 9 (Pages 142–143) to do.

Angles on the alternate segments of a circle (Page 143)

Lead the students through and explain Example 12. State Theorems 4–7. Lead the students to draw the diagram and guide them to carry out the formal proof of these theorems; explain Example 15 first, then solve practical problems on them. Ensure that the students solve all questions in Exercise 11.4 (Pages 144-146). Let the students study the theorems step-by-step. This will enable them to understand the theorems and when to use them. Give them Exercise 11.5 (Pages 150 and 151) as an assignment and Exercise 11.6. (Page 151 and 152) as class test.

Chapter 12 Trigonometry: Sine and cosine rules

Objectives

By the end of this chapter, the students will be able to:

1. derive the sine rule using the acute and obtuse-angled triangle;
2. solve problems on triangles using the sine rule;
3. derive the cosine rule using the acute and obtuse-angled triangle;
4. solve problems on triangles using the cosine rule; and
5. apply the sine and cosine rule in solving triangles and other related problems.

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Trigonometrical ratios (Revision)

Lead the students to define the terms SOH, CAH, TOA.

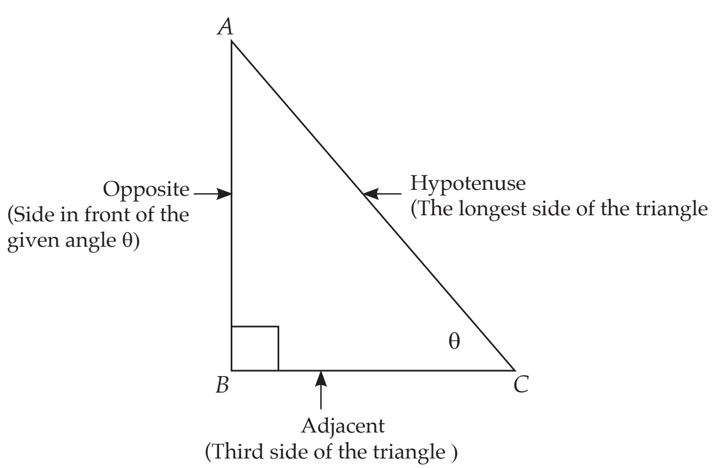
SOH means $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$;

CAH means $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$; and

TOA means $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$.

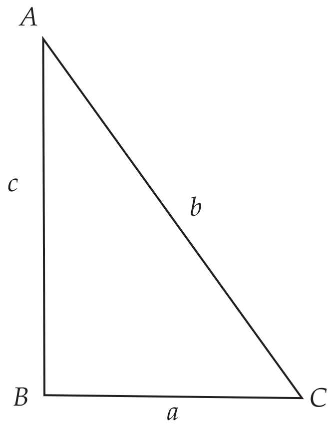
Guide the students to understand that sine, cosine and tangent of an acute-angled triangle are used in solving problems on right-angled triangles.

Display a chart showing acute angle and obtuse angle. Ask the students to define acute and obtuse



angles and give examples of each.

Lead the students to use the chart to explain the conventional methods of naming the vertices and sides of triangle.



Capital letters are used for vertices of a triangle, while small letters are used for sides of a triangle.

Side of vertex A is $|BC| = a$

Side of vertex B is $|AC| = b$

Side of vertex C is $|AB| = c$

Revise the trigonometrical ratios of angles from 0° to 360°.

y

2nd quadrant $(180 - \theta) \Leftarrow 90° < \theta < 180°$ *Only sine is positive*	**1st quadrant** $0° < \theta < 90° \Rightarrow \theta$ *All the ratios are positive*
3rd quadrant $(\theta - 180) \Leftarrow 180° < \theta < 270°$ *Only tangent is positive*	**4th quadrant** $270° < \theta < 360° \Rightarrow (360° - \theta)$ *Only cosine is positive*

The sine rule and its application (Page 153)

Consider the acute and obtuse angles in Fig. 12. Guide the students to prove the sine rule and arrive at the expression $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Give the students classwork on Questions 1, 2 and 3 of Exercise 12.1 (Page 155).

Lead them to Example 1 (Page 155) by explaining to them for easy understanding.

Guide the students on how to apply the sine rule in finding solutions to triangles that are not right-angled (i.e. acute-angled triangle and obtuse-angled triangle).

The cosine rule and its application (Page 155)

Guide the students to derive expressions:

a) $a^2 = b^2 + c^2 - 2bc \cos A$

b) $b^2 = a^2 + c^2 - 2ac \cos B$

c) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

d) $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

e) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Let the students realise that cosine rule is used for solving triangles in which

i) the two sides and one included angle are given.

ii) the three sides are given.

Use Examples 2–7 (Page 157) to explain how to apply cosine rule in solving problems on triangle. Ensure that the students attempt Questions 1, 2 and 3 in Exercise 12.2 (Page 158). Inspect their work and make necessary corrections.

Application of sine and cosine rules to other related problems (Page 158)

Guide them to problems on angle of elevation and depression, make reference to the workbook. Treat Examples 3 and 4 with them. Give them Questions on 1, 3, 5, 6, 7 and 8 of Exercise 12.3 (Pages 160 and 161) as classwork.

Lead the students to use Pythagoras theorem to obtain two expressions for h in terms of a, b and x. Guide the students to deduce expression for

$\cos C = \dfrac{x}{a} \Rightarrow x = a \cos C.$

Give the students all the questions in Exercise 12.4 (pages 161 and 162) as assignment.

Chapter 13 Bearings

Objectives

By the end of this chapter, the students will be able to:

1 solve problems on angle of elevation and depression using trigonometrical ratios;

2 define four, eight and sixteen-cardinal points;

3 state the two bearing notations which involve acute bearing (N50°W) and three-figure bearing (050°);

4 sketch the diagrams and solve problems involving bearing, using any of trigonometrical ratios, sine rule or cosine rule; and

5 solve practical problems on bearing.

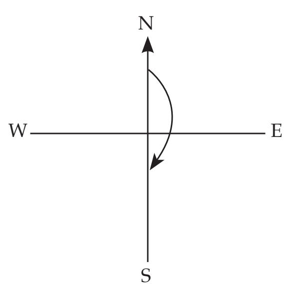
Angles of elevations and depression (Page 170)

Lead the students on how to draw angles of elevation and depression. Discuss Examples 1–4 (Pages 171–172) with them and encourage them to attempt all the previous *WAEC* questions in Exercise 13.1 (Pages 172 and 174) in the class and the rest can be given as homework.

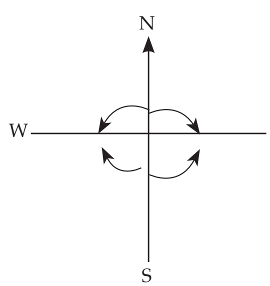
The cardinal points (Page 174)

Use Figs 13.8, 13.9 and 13.10 (Pages 174 and 175) to explain four and eight-cardinal points and define bearing.

Bearing is the angle between a line and a reference direction, often measured clockwise from the North (if it is three-figure) or through cardinal points. N, S, E, W (if it is an acute angle). The movement is either from North towards either West or East or from the South towards either West or East.



Three-figure bearing



Cardinal point movement

Let the students understand that accurate interpretation is highly recommended in solving bearings and distances problems. Lead the students to determine when to use Pythagoras theorem, trigonometric ratios, sine and cosine to solve bearing problems, using Examples 5–6 (Pages 176–177). Discuss Exercise 13.2 (Pages 177) with the students. This will help them to understand the topic better. Encourage them to interpret all the questions and give them all the previous *WAEC* questions to do as assignment.

Practical problems on bearings and distances (Page 178)

Guide them to diagram sketching under this topic. The sketching should always be followed by solving the various components required for. Let the students know that sketching is very important under this topic. Any explanation should always include sketch.

Guide them through Examples 7 – 10 (pages 178 –179) and give them Questions 1 – 10 of Exercise 13.3 (pages 179 – 181) as classwork and Question 11 – 20 as assignment.

Chapter 14

Histograms of grouped data

Objectives

By the end of this chapter, the students will be able to:

1 appreciate the need for grouping data;

2 construct grouped frequency tables; and

3 calculate class intervals, limit boundaries and marks.

Grouping of data (Page 182)

Explain to the students that data is the collection of raw information on some characteristics of a group of individuals or objects under study. Sometimes, these information gathered may be so large and complicated. The easiest way to handle these bulky data and make them simple to understand is by organising them and this process of organising these data is known as grouping data. Grouped data in statistics is any data collected together for a particular purpose. Use Example 1 (Page 183) to explain how to prepare data using tally method to construct frequency table.

Lead the students to define class interval, class limit (i.e. lower and upper class limit) and class boundary. Let the students realise that class boundary of a class interval can be found by subtracting 0.5 from lower class limit and by adding 0.5 to upper class limit. Guide the students to determine class mark.

$$\text{Class mark} = \frac{\text{lower class limit} + \text{upper class limit}}{2}$$

Discuss Questions 5 (Page 184) in Exercise 14.1 (Pages 184–185) with the students in the classroom. Encourage them to solve Questions 7, 8 and 9 in the same exercise.

Histogram (Page 185)

Define histogram and describe how to construct histogram of a given data using Examples 4 and 5 (Pages 185 and 186). Discuss Question 7 in Exercise 14.2 (Pages 186–188) with the students and ask them to write out the solution in their note. Ensure that they attempt Question 8 in the same exercise (Page 188). Later go through their work in order to make necessary corrections.

Chapter 15 Measures of central tendency and dispersion of disctrete data

Objectives

By the end of this chapter, the students will be able to:

1 define and calculate the mean, median and mode of ungrouped data;

2 define and calculate the range, variance and standard deviation of a set of data; and

3 apply (1) and (2) to daily activities like tests and population.

Measures of central tendency (Page 189)

Lead the students to define mean, median and mode, and write out the formulae for calculating mean of ungrouped data.

The arithmetic mean (Page 189)

Let the students realise that there are many ways of calculating mean of ungrouped data:
We can use

$\bar{x} = \dfrac{\sum x}{n}$ where $\sum x$ means the summation of the values or scores.

$\bar{x}$ = mean

n = number of terms in the given data

or $\bar{x} = \dfrac{\sum fx}{\sum f}$ for ungrouped data with frequency.

Where f = frequency

x = values or scores

$\sum fx$ = summation of frequency and scores

$\sum f$ = total frequencies or summation of frequency

or

$$\bar{x} = A + \frac{\sum d}{n}$$

where A = is the assumed mean

d = deviation from mean

$\sum d$ = summation of deviation from mean

n = number of terms in the given data

Use Examples 1 to 3 (Pages 189 and 190) to explain how to use the three formulae to calculate the mean of ungrouped data.

The median (Page 190)

The median of a set of n members is defined as $\frac{1}{2}(n+1)$ th value when the n members are arranged in the order of magnitude. Discuss Examples 4–6 (page 191) and Questions 5, 7, 8, 9 and 10 of Exercise 15.1 (Pages 192 and 193) with the students in class. Encourage them to solve the questions discussed in their notebooks.

The mode (Page 191)

The mode is the value that occurs most frequently in a set of data. It has the highest frequency. Discuss Examples 7 and 8 (Pages 191 and 192) with the students.

Mixtures and average rate (Page 193)

Guide the students through Example 9 and 10 (page 193) and give them all the WAEC past questions in Exercise 15.2 (page 194) as classwork and the rest as assignment.

Measures of dispersion (Page 194)

Lead the students to define the range variance and standard deviation.

The range is the difference between the highest number and the lowest number. For example: the range of 5, 14, 2, 11, 8, 2 and 15 is the difference between 15, which is the highest number and 2, which is the lowest number.

$\therefore$ the range = 15 – 2 = 13

The variance is given by $\dfrac{\sum(x-x)^2}{n}$ $\rightarrow$ for ungrouped discrete.

where x is the value or score and $\bar{x}$ is the calculated mean $\left(\text{i.e.} \dfrac{\sum x}{n}\right)$

$\Sigma(x-\bar{x})$ = deviation of x from the ($\bar{x}$)

n = number of terms or total number;

or

variance $= \dfrac{\sum f(x-\bar{x})^2}{\sum f}$ $\rightarrow$ for ungrouped discrete data with frequency.

Standard deviation is the positive square root of variance which is given by

S.D $= \sqrt{\dfrac{\sum(x-\bar{x})^2}{n}}$ $\rightarrow$ for ungrouped data

or S.D $= \sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}}$ $\rightarrow$ for ungrouped data with frequency

$$\text{Variance} = \frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2 \rightarrow \text{for grouped data.}$$

$$\text{S.D} = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} \rightarrow \text{for grouped data.}$$

Use Examples 11 to 15 (Pages 195–197) to explain how to apply the formulae to calculate variance and standard deviation of ungrouped and grouped data. Discuss Exercise 15.3, Questions 2, 3, and 4 (Page 197) with the students. Ensure that the students attempt Questions 1, 5 and 6 in the same exercise (Page 197).

Chapter 16 Measures of central tendency and dispersion of grouped data

Objectives

By the end of this chapter, the students will be able to:

1 calculate the mean of a grouped data using
- arithmetic mean method;
- the assumed mean method;

2 find the median of a grouped data;

3 determine the mode of a grouped data; and

4 calculate the variance and standard deviation of group data.

Measures of central tendency (Page 198)

Explain to the students that when data is very large, it is advisable to group them in order to make analysis easier to interpret.

Guide the students on the steps to follow when numerical data is very large. Discuss:

1 class interval;

2 class limit;

3 class boundary; and

4 class mark.

Mean of grouped data (Page 198)

Lead the students to Example 1 (Page 198) for easy understanding and explanation. Lead them through Tables 16.1–16.3 for easy understanding.

Lead the students to Examples 2 and 3 (Pages 198 and 199) for the mean of group and ungrouped data (indicate the differences between the two).

Guide the students on how to use the **assumed mean method** and **arithmetic mean method**.

$$\text{Mean}\left(\bar{x}\right) = \frac{\sum fx}{\sum f} \quad - \text{ Arithmetic mean method}$$

$$\text{Mean}\left(\bar{x}\right) = A + \frac{\sum fd}{\sum f} \quad - \text{ Assumed mean method.}$$

Explain the following symbols:

A = assumed mean

d = deviation from assumed mean

f = frequency

$\bar{x}$ = arithmetic mean

Lead the students to Example 4 (Page 200) and explain how the assumed mean method is used in getting or calculating the mean. Guide the students to understand that sometimes only small difference occurs between the two methods.

Median of grouped data (Page 200)

Lead the students on how to calculate the median of a grouped data using the formula below.

$$\text{Median} = L + \left(\frac{\frac{n}{2} - \Delta_1}{f} \right) \times c$$

Explain the symbols

L = lower class boundary of the median class

Median class = $\frac{n+1}{2}$

n = number of data

$\triangle_1$ = cumulative frequency before the median class

f = frequency of median class

c = class size

Lead them to Example 5 (Page 201) for easy explanation and understanding.

Mode of grouped data (Page 201)

Lead the students on how to calculate the mode using the formula

$$\text{Mode} = L_1 + \left(\frac{d_1}{d_1 + d_2} \right) \times c$$

Explain the symbols, see page 201.
Lead them to Examples 6 and 7 (Pages 201 and 202) for easy understanding.

Guide the students to work the following questions in the class. Exercise 16.1 (Pages 202–204) Questions 2, 3 and 5 (Page 203). Supervise them in the class, then, mark and give the necessary corrections.
Give the students Questions 4, 6, 7, 8, 9 and 10 as assignment.

Measures of dispersion (Page 204)

Lead the students to Examples 8 and 9 (Pages 205–206) for easy explanation and understanding. Lead them on how to plot smooth curve and how to use the curve to answer questions (median and quartiles etc).

Exercise 16.2 (Pages 207 and 208)

Lead the students to answer Questions 1, 3 and 4 (Page 207) of the above exercise as classwork. Mark and give the necessary corrections.

Chapter 17

Cumulative frequency graph

Objectives

By the end of this chapter, the students will be able to:

1 compute the cumulative frequency table of grouped data;
2 draw the cumulative frequency curve (known as Ogive curve) of grouped data;
3 make estimates from the Ogive curve; and
4 apply cumulative frequency curve to real-life situations.

Cumulative frequency (Page 209)

Revise how to construct frequency table with the students. Then, explain steps involved in constructing cumulative frequency table in the class.
Cumulative frequency table can be found by adding the frequencies as shown in the table below.

Mark	Frequency (F)	Class boundary	Cumulative frequency (CF)
41 – 50	6	40.5 – 50.5	6
51 – 60	7	50.5 – 60.5	6 + 7 = 13
61 – 70	10	60.5 – 70.5	13 + 10 = 23
71 – 80	6	70.5 – 80.5	23 + 6 = 29
81 – 90	3	80.5 – 90.5	29 + 3 = 32
	32		

From the above table, the total frequency is 32 on the frequency column, this total should correspond to the final cumulative frequency on the cumulative frequency column.

Guide the students to make the class intervals continuous as shown in the table on the class boundary column.
Lead the students on how to draw the graph. The vertical axis is the cumulative frequency and the horizontal axis is the upper class boundaries of the group intervals. Let the students understand that a well scaled graph and smooth curve will always produce a good or accurate results.

Lead the students to draw the cumulative frequency curve using the upper-class boundaries.
Lead the students to define the following terms, using Examples 1 and 2 (Page 210)

Quartiles and percentiles (Page 212)

a) Quartiles:

Lower quartile (Q_1) $\rightarrow$ Given by $\frac{1}{4}$ (N + 1)th

Upper quartile (Q_3) $\rightarrow$ Given by $\frac{3}{4}$ (N + 1)th

Medium Q_2 or (M) $\rightarrow$ Given by $\frac{1}{2}$ (N + 1)th

Interquartile range $\rightarrow Q_3 - Q_1$

Semi-quartile range $\rightarrow \frac{Q_3 - Q_1}{2}$ etc.

b) Percentile: this divides the distribution into hundred parts.

Lead the students to understand that the 50th percentile is the same as the median. Interpret and explain how to use the cumulative frequency curve to estimate quartiles, percentiles, etc.
Lead the students to understand that the cumulative frequency is arranged from smallest (least) to the highest (largest) value or arranged in ascending order from the bottom of the vertical axis to the top.

Lead the students to Example 4 (Pages 213–214) for easy understanding.

Cumulative percentage curve (Page 216)

Lead the students to cumulative percentage curve by making reference to Example 6 (Pages 216 and 217) for easy explanation and understanding.
Guide the students to the difference between cumulative percentage (CP) and cumulative frequency.

Lead them to know how to plot the curve and how to use it to find a centile point, related to the horizontal axis.
See Example 6 (Page 216) on how the table is constructed (Table 17.10) and graph plotted. Give the students questions in Chapter 17, Exercise 17 (Pages 217 and 223) Questions 2 and 4.

Chapter 18 Probability

Give the rest as an assignment.

Objectives

By the end of this chapter, the students will be able to:

1 define and explain the concept of probability;
2 define and explain terms used in probability;
3 give practical examples of each term;
4 describe major approaches to probability;
5 estimate the probability of events by carrying out experiments;
6 calculate the probability for equally likely events;
7 describe and solve problems in mutually exclusive events;
8 solve problems on independent, complementary independent events;
9 correctly apply addition law to find the probability of mutually exclusive events;
10 correctly apply the multiplication law to find the probability of independent events;
11 solve problems on probability experiment with or without replacement; and
12 solve problems on probability using Venn diagrams and probability tree.

Introduction

Explain to the students that probability can be defined as a branch of mathematics that deals with the possibilities or chances of events happening under given conditions, or the mathematical expression of the extent to which an event is likely to occur, given a value between 0 (an impossibility) and 1 (a certainty). There are two types of probability namely:

a) experimental probability
b) theoretical probability

Experimental and theoretical probability (Page 224)

Experimental probability bases its result on the actual experiment carried out, and the outcome will therefore, be based on the number of attempts made. Experimental probability uses past events to predict the future.

Theoretical probability bases its results on exact values that are depending on the physical nature of the situations under consideration.

Use Tables 18.1 –18.3 (Page 225) to explain possible outcomes of throwing a pair of fair dice once and Table 18.4 to explain possible outcomes of playing cards, colours and symbols.

Discuss Examples 1 and 2 (Pages 224 and 225) with the students. Give them Questions 5 and 6 in Exercise 18.1 (Pages 226 and 227) to do as their assignment, then encourage them to solve Questions 8–10 in the same exercise in the classroom.

Use Examples 5–7 (Pages 228 and 229) to explain theoretical probability. Ensure that the students attempt Questions 5, 6, 8, 9 and 10 in Exercise 18.2 (Pages 231 and 232) and discuss any difficult question with them in the classroom.

Mutually exclusive events (Page 232)

Lead the students to define mutually exclusive independent and complementary events. Two events are said to be mutually exclusive when one event prevents the occurrence of another i.e. the two events can not happen at the same time. For example, it is impossible to throw a dice and score a three and a four at the same throw with a single dice. Mutually exclusive event can be referred to as addition of probability.

$\text{Pr}\ (X \text{ or } Y) = \text{Pr}\ (X) + \text{Pr}\ (Y)$

Two events are complementary or independent if they can occur without affecting each other. Complementary or independent events are also referred to as multiplication of probability. Words like *and*, *both* and *all* are used in independent events. Probability of X and Y occurring is given by

$\text{Pr}\ (X \text{ and } Y) = \text{Pr}\ (X) \times \text{Pr}\ (Y)$

Discuss Examples 11 and 12 (Page 233) with the students to explain mutually exclusive and complementary or independent events respectively. Discuss all the difficult questions in Exercise 18.3 (Page 233) and (Pages 235 and 236) and 18.4 with the students in classroom, then encourage them to attempt at least five questions from each exercise.

Independent events (Page 236)

Use Examples 14–17 (Pages 236–237) to explain further on complementary/independent events and discuss all the previous WAEC questions in Exercise 18.5 (Pages 239 and 240) with students in the classroom. Encourage them to write out the questions on the board.

Probability tree (Page 240)

Lead the students to discover how to use tree diagrams to solve probability problems using Examples 20 and 21 (Pages 241 and 242). Encourage the students to attempt Questions 6–10 of Exercise 18.6 (Pages 242 and 243) and discuss any difficult questions with them in the classroom.

Hints:

a) Addition law is used to solve problems that contain the word *or*
e.g. probability of X or Y is given by
$\text{Pr}\ (X \text{ or } Y) = \text{Pr}\ (X) + (Y)$

b) Probability of getting a head or a tail in tossing a coin at once will be

$$\Pr(H \text{ or } T) = \Pr(H) \text{ or } \Pr(T)$$

$$\Pr(H) = \frac{1}{2}$$

$$\Pr(T) = \frac{1}{2}$$

$$\therefore \Pr(H \text{ or } T) = \frac{1}{2} + \frac{1}{2} = 1$$

c) Product law is used to solve problems that contain the word *and* or *both* with *and e.g.* probability of head and tail in tossing a coin at once:

$$\Pr(H \text{ and } T) = \frac{1}{2},\frac{1}{2} = \frac{1}{2}$$